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**Investigation of Burnett Equations
for Two-Dimensional Hypersonic Flow**

Submitted to the
Air Force Office of Scientific Research
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November 1991

INVESTIGATION OF BURNETT EQUATIONS FOR TWO-DIMENSIONAL HYPERSONIC FLOW

ABSTRACT

Our research to date on the Burnett equations has identified five basic scientific issues in need of resolution before really satisfactory computations of 2D (or 3D) flow fields can be made with these equations. Briefly, these issues relate to (1) surface boundary conditions, (2) frame independence, (3) material derivative approximation, (4) positive-definite dissipation (?), and (5) upper altitude limit for applicability.

The progress reported on herein involves three different areas of investigation conducted by three different research assistants: (A) research on issues (1) and (4); (B) research on issue (3); and (C) investigation of the interaction of a thick oblique shock impinging on a cowl lip in high-altitude hypersonic flow.

(A). Issue (1)—Computations of 2D hypersonic flow over a flat plate have shown that the question of what surface slip boundary conditions are most physically realistic for use with the Burnett equations, is a crucial one. Comparison with corresponding particle-flow simulations reveals that the conventional Maxwell/Smoluchowski boundary conditions for velocity slip and temperature jump are not adequately realistic at medium and high Knudsen numbers. Better slip and jump conditions are needed.

(A). Issue (4)—Burnett computations of the dissipation function for 2D hypersonic flow over a blunt leading edge show the dissipation to be everywhere positive, suggesting that this particular issue may not be a troublesome one.

(B). Issue (3)—Exploration has begun on different forms of the Burnett equations for 1D hypersonic shock structure using various improved approximations for material derivative (improved relative to the approximation conventionally made), and no approximation

at all. Each form involves different physical terms and introduces different numerical computation difficulties, especially in the Burnett energy conservation equation. Results to date on analytical stability analyses are outlined.

(C).—An analysis has been completed of the means for generating, from either the Navier-Stokes or Burnett equations, the velocity, density, and temperature field within an oblique shock wave structure. Such generation is necessary for specifying the outer computational boundary condition that will produce a thick, oblique, impinging shock wave. It is found that the velocity component in the direction parallel to the oblique shock wave is everywhere constant within the structure of that wave. This analytical result greatly simplifies the procedure for setting up numerical computations of the interacting flow field of a thick oblique shock impinging on a cowl lip.

A. PROGRESS ON THE SURFACE BOUNDARY CONDITION AND THE DISSIPATION ISSUES

Progress in this area was made between October 1, 1990 and August 1, 1991, during which period Xiaolin Zhong was a research assistant. He has completed his Ph.D. thesis, and has joined the faculty at UCLA on August 1, 1991. Research results were presented in a paper given at the 4th International Symposium on Computational Fluid Dynamics, U. C. Davis, September 9-12, 1991, entitled "Evaluation of Slip Boundary Conditions for the Burnett Equations With Application to Hypersonic Leading Edge Flows" by Xiaolin Zhong, Robert W. MacCormack, and Dean R. Chapman. A copy of this paper is appended to the present annual report. The main results concerning surface boundary conditions are briefly summarized in the *Abstract* above, and are described in more detail in the appended paper.

Our results to date concerning the issue of whether the Burnett dissipation is positive definite are illustrated in Figure 13, the last figure in this report. The Burnett dissipation is everywhere positive in this 2D hypersonic flow field in front of a blunt leading edge, as required by physical considerations. Most of this dissipation comes from the Navier-Stokes terms, as might be expected. We have no indication thus far of any problem with dissipation using the Burnett equations.

B. Development of Alternate Burnett Constitutive Equations

Introduction

During the past 55 years, a number of researchers have studied and sought to find practical solutions to the Burnett equations. The clear majority of research, especially attempts at numerical solutions, have been accomplished using various forms of what will be termed the Conventional Burnett equations. At Stanford, the recent advances in obtaining numerical solutions also made use of different forms of the Conventional Burnett equations. Yet, it is in the derivation of these forms of the Conventional Burnett equations that we may gain insight into a potentially more accurate form of the Burnett constitutive relations.

In 1935, Burnett^[1] developed a higher order set of constitutive stress relationships from a class of solutions to the Boltzmann equations. An example of one term of the original Burnett Stress', which is added to the Navier-Stokes stress, is listed below.

$$\begin{aligned}
 \sigma_{zz} = & \frac{2}{3} K_1 \mu \frac{D}{Dt} \frac{\mu}{p} \left(2 \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) - K_1 \frac{\mu^2}{p} \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 - \left(\frac{\partial w}{\partial x} \right)^2 - \left(\frac{\partial w}{\partial y} \right)^2 \right) \\
 & - K_2 \frac{\mu^2}{v h m p} \left(2 \frac{\partial h}{\partial z} \frac{\partial p}{\partial z} - \frac{\partial h}{\partial x} \frac{\partial p}{\partial x} - \frac{\partial h}{\partial y} \frac{\partial p}{\partial y} \right) \\
 & - K_3 \frac{\mu h}{v m} \left(2 \frac{\partial}{\partial z} \frac{\mu}{h^2} \frac{\partial h}{\partial z} - \frac{\partial}{\partial x} \frac{\mu}{h^2} \frac{\partial h}{\partial x} - \frac{\partial}{\partial y} \frac{\mu}{h^2} \frac{\partial h}{\partial y} \right) \\
 & - K_4 \frac{\mu^2}{v m h^2} \left(2 \left(\frac{\partial h}{\partial z} \right)^2 - \left(\frac{\partial h}{\partial x} \right)^2 - \left(\frac{\partial h}{\partial y} \right)^2 \right) \\
 & + K_5 \frac{\mu^2}{9 p} \left(2 \left(2 \frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)^2 + 3 \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + 3 \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right. \\
 & \left. - 6 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 - 6 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \right)
 \end{aligned} \tag{1}$$

¹ In his paper, Burnett used the z-axis as the principal flow direction.

Note the presence of the material derivative* in the first term of the stress expression. This derivative also appears in the other five stress terms.

In 1939, Chapman and Cowling^[2] published an alternate form of the Burnett stress term as well as the corresponding order heat conduction expression. The publication developed forms of the Burnett stress and heat conduction terms where the material derivative terms were replaced with spatial flow field gradients through the use of the inviscid conservation equations for momentum and energy. These forms of Burnett equations we here to refer as the "Conventional" Burnett equations. Replacing the material derivatives eliminates the time dependent terms in the Burnett constitutive relationships in favor of partial derivatives of the same order as the remaining Burnett stress and heat conduction terms.

Sporadic research into Burnett applications transpired over the next several decades with limited success in finding practical solutions. Research during this period centered on the Conventional Burnett equations. In 1948, Wang Chang and Uhlenbeck^[3] reexamined the Conventional Burnett equations, attempting to find solutions using a type of series approximation. In 1959, Talbot and Sherman^[4] were able to obtain solutions to the Burnett equations for Mach numbers below 2. In 1973, Foch^[5] attempted to use an ordinary differential equation approach to numerically solve the Conventional Burnett equations, but was unable to obtain solutions for Mach numbers above what Talbot and Sherman had achieved more than a decade earlier. In 1976, Tannehill and Eisler^[6] examined the Conventional Burnett equations for a flow over a leading edge.

In the early 80s, Woods^[7,8] developed the Burnett equations in a derivation independent of the earlier Chapman-Enskog expansion method Burnett used in 1935. Instead, Woods used mean free path arguments to obtain the stress and heat conduction relationships. Interestingly, the constitutive relationships contained a material derivative term in the stress and heat conduction expressions. The Woods formulated Burnett expressions are

* The material derivative has the formal definition: $D = \frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x_j}$.

$$\begin{aligned}
\bar{Q}_{Woods} &= \frac{\mu^2}{p} \left\{ \omega_1 \overset{\times}{e} \overset{\circ}{e} + \omega_2 \left(D \overset{\circ}{e} - 2 \Omega \times \overset{\circ}{e} \right) + \omega_3 \overset{\circ}{R} \nabla \nabla T + \omega_4 \overset{\circ}{R} \nabla P \nabla T \right. \\
&\quad \left. + \omega_5 \overset{\circ}{R} \nabla T \nabla T + \omega_6 \overset{\circ}{e} \cdot \overset{\circ}{e} \right\} \\
\bar{q}_{Woods} &= \frac{\mu^2}{\rho T} \left\{ \theta_1 \overset{\times}{e} \nabla T + \theta_2 (D \nabla T - \Omega \times \nabla T) + \theta_3 \overset{\circ}{T} \overset{\circ}{e} \cdot \nabla p + \theta_4 T \nabla \cdot \overset{\circ}{e} + \theta_5 \overset{\circ}{e} \cdot \nabla T \right\}
\end{aligned} \tag{2}$$

In the general Burnett expressions listed above, $\overset{\circ}{e}$ is the deviator of the velocity gradient tensor, and is related to the velocity gradient tensor by the expression

$$\overset{\circ}{e} = \frac{1}{2} (\mathbf{e} + \tilde{\mathbf{e}}) - \frac{1}{3} \overset{\times}{e} \mathbf{I}$$

where: $\mathbf{e} \equiv \nabla \mathbf{V}$
 $\tilde{\mathbf{e}} \equiv \text{Transpose of } \mathbf{e}$
 $\overset{\times}{e} \equiv \text{Trace of } \mathbf{e}$
 $\mathbf{I} \equiv \text{Identity Tensor}$

Given a tensor \mathbf{A} , the operator $\overset{\circ}{\mathbf{A}}$ has the formal definition

$$\overset{\circ}{\mathbf{A}} = \frac{1}{2} (\mathbf{A} + \tilde{\mathbf{A}}) - \frac{1}{3} \mathbf{A} : \mathbf{I}$$

where: $\tilde{\mathbf{A}} \equiv \text{Transpose of } \mathbf{A}$
 $\mathbf{I} \equiv \text{Identity Tensor}$

Research into the Burnett equations at Stanford began in the mid-1980s. Fisko^[9] investigated two forms of the Conventional Burnett equations for the shock structure of monatomic gases. Lumpkin^[10] followed the work of Fisko by extending the applicability of the Burnett equations to polyatomic gases. Lastly, Zhong^[11] solved a stability problem of the Conventional Burnett equations by adding three Super-Burnett like terms to obtain the augmented Burnett equations. These allowed numerical solutions of the Burnett equations to be computed for any Mach number at any altitude.

The Conventional Burnett Equations

A closer examination of the Conventional Burnett equations is best carried out in one-dimension. Recall from Woods the stress and heat conduction terms which contain the material derivative. These may be written:

$$\begin{aligned}\sigma_{Woods\ Burnett} &= \frac{\mu^2}{p} \left(\dots + Const \frac{D}{Dt} \left(\frac{\partial u}{\partial x} \right) + \dots \right) \\ q_{Woods\ Burnett} &= \frac{\mu^2}{\rho T} \left(\dots + Const \frac{D}{Dt} \left(\frac{\partial T}{\partial x} \right) + \dots \right)\end{aligned}\quad (3)$$

The material derivative in the stress term may be approximated as

$$\frac{D}{Dt} \left(\frac{\partial u}{\partial x} \right) \Rightarrow - \frac{\partial}{\partial x} \left(\frac{1}{\rho} \frac{\partial p}{\partial x} \right) - \left(\frac{\partial u}{\partial x} \right)^2, \quad (4)$$

using the inviscid momentum equation

$$\frac{Du}{Dt} = - \frac{1}{\rho} \frac{\partial p}{\partial x}. \quad (5)$$

The material derivative in the heat conduction term may be approximated as

$$\frac{D}{Dt} \left(\frac{\partial T}{\partial x} \right) \Rightarrow - \frac{\partial}{\partial x} \left(\frac{p}{\rho c_v} \frac{\partial u}{\partial x} \right) - \frac{\partial u}{\partial x} \frac{\partial T}{\partial x}, \quad (6)$$

using the inviscid energy equation

$$\frac{DT}{Dt} = - \frac{p}{\rho c_v} \frac{\partial u}{\partial x}. \quad (7)$$

These Conventional Burnett equations become in one dimension

$$\begin{aligned}\sigma_{Conventional\ Burnett} &= \frac{\mu^2}{p} \left(\dots + Const \left[- \frac{\partial}{\partial x} \left(\frac{1}{\rho} \frac{\partial p}{\partial x} \right) - \left(\frac{\partial u}{\partial x} \right)^2 \right] + \dots \right) \\ q_{Conventional\ Burnett} &= \frac{\mu^2}{\rho T} \left(\dots + Const \left[- \frac{\partial}{\partial x} \left(\frac{p}{\rho c_v} \frac{\partial u}{\partial x} \right) - \frac{\partial u}{\partial x} \frac{\partial T}{\partial x} \right] + \dots \right)\end{aligned}\quad (8)$$

The substitution made in forming the Conventional Burnett equations is due to an approximation. For regions of a flow field where gradients are reasonably small, the approximation made in writing the Conventional Burnett equations is a good one. When flow field gradients are large, in a boundary layer, shock, or a free shear layer, the

approximation made in developing the Conventional Burnett expressions must be examined more closely.

Recall that the substitutions made for the material derivative terms were completed through the use of inviscid conservation equations for momentum and energy. A comparison was made of the left and right hand side terms of equations 5 and 7 through a normal shock in Argon gas as computed from the Navier-Stokes equations. Figure 1 plots the inviscid momentum terms and figure 2 plots the inviscid energy equation terms, each non-dimensionalized by the appropriate free-stream quantities and an appropriate mean free path.

If the approximations made in developing the respective Conventional Burnett stress and heat conduction terms are accurate then the material derivative terms, represented by a solid line in figures 1 and 2, should be close to the approximated terms, represented by a dashed line. A comparison of the respective stress terms in figure 1 shows a fairly good match between the two expressions. The primary difference is that the material derivative expression is shifted upstream of the pressure gradient expression. The differences in figure 2, corresponding to the heat conduction terms, are much more striking. While the material derivative expression is shifted upstream from the velocity gradient expression as was the case for the stress comparison, here the magnitude of the material derivative term is a factor of two larger. The approximation used in developing the heat conduction expression of the Conventional Burnett equations introduces a sizeable error when gradients are significant. This observation apparently has not been made before.

Development of Alternate Expressions

Two ideas are readily apparent to overcome the error introduced when approximating the material derivative in the Burnett stress and heat conduction expressions. The first idea, and most obvious, is to keep the material derivative expression intact, without approximations, when developing the constitutive relationships for stress and heat conduction. This leads to what we term the Material Derivative Based (MDB) Burnett equations. The second idea is to use the viscous momentum and energy equations (Navier-Stokes) instead of the inviscid momentum and energy equations when rewriting the material derivative terms in the Burnett stress and heat conduction expressions. This

leads to what we term the Navier-Stokes Based (NSB) Burnett equations. Upon introducing the two ideas, the two alternate sets of Burnett equations are easily developed.

It is useful to examine the different forms of the Burnett equations in one-dimension for a Maxwellian gas. The stress and heat conduction terms for the Conventional Burnett equations are:

$$\begin{aligned} \sigma_{\text{Conventional Burnett}} = & -\frac{4}{3} \mu \frac{\partial u}{\partial x} + \frac{\mu^2}{\rho} \left[\frac{8}{9} \left(\frac{\partial u}{\partial x} \right)^2 - \frac{4}{3} \frac{RT}{\rho} \frac{\partial^2 \rho}{\partial x^2} + \frac{4}{3} \frac{RT}{\rho^2} \left(\frac{\partial \rho}{\partial x} \right)^2 - \frac{4}{3} \frac{R}{\rho} \frac{\partial \rho}{\partial x} \frac{\partial T}{\partial x} \right. \\ & \left. + 2 \frac{R}{T} \left(\frac{\partial T}{\partial x} \right)^2 + \frac{2}{3} R \frac{\partial^2 T}{\partial x^2} \right] \end{aligned} \quad (9)$$

$$q_{\text{Conventional Burnett}} = -k \frac{\partial T}{\partial x} + \frac{\mu^2}{\rho} \left[\frac{95}{8} \frac{\partial u}{\partial x} \frac{\partial T}{\partial x} - \frac{7}{4} \frac{\partial^2 u}{\partial x^2} - \frac{2}{\rho} \frac{\partial u}{\partial x} \frac{\partial \rho}{\partial x} \right]$$

The stress and heat conduction terms of the material derivative based (MDB) Burnett equations, where the original definition of the material derivative is maintained in developing the Burnett expressions, are:

$$\begin{aligned} \sigma_{\text{MDB}} = & -\frac{4}{3} \mu \frac{\partial u}{\partial x} + \frac{\mu^2}{\rho} \left[\frac{4}{3} \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} \right) + \frac{4}{3} \mu \frac{\partial^2 u}{\partial x^2} + \frac{20}{9} \left(\frac{\partial u}{\partial x} \right)^2 + 2 \frac{R}{T} \left(\frac{\partial T}{\partial x} \right)^2 + 2 R \frac{\partial^2 T}{\partial x^2} \right] \\ q_{\text{MDB}} = & -k \frac{\partial T}{\partial x} + \frac{\mu^2}{\rho} \left[\frac{45}{8} \frac{\partial}{\partial t} \left(\frac{\partial T}{\partial x} \right) + \frac{45}{8} \mu \frac{\partial^2 T}{\partial x^2} + \frac{85}{4} \frac{\partial u}{\partial x} \frac{\partial T}{\partial x} + 2 \frac{\partial^2 u}{\partial x^2} - \frac{2}{\rho} \frac{\partial u}{\partial x} \frac{\partial \rho}{\partial x} \right] \end{aligned} \quad (10)$$

Since these equations for one-dimensional flow are equivalent to the corresponding equations derived by Burnett in 1935 for a one-dimensional flow, they also could be termed the "Original" Burnett equations.

The stress and heat conduction terms of the Navier-Stokes based (NSB) Burnett equations, where the viscous momentum and energy equations are used to rewrite the material derivative expressions, are:

$$\begin{aligned}
\sigma_{NSB} &= \sigma_{\text{Conventional Burnett}} + \frac{\mu^2}{\rho} \left[\frac{16}{9} \frac{1}{\rho} \frac{\partial^2}{\partial x^2} \left(\mu \frac{\partial u}{\partial x} \right) - \frac{16}{9} \frac{1}{\rho^2} \frac{\partial \rho}{\partial x} \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial x} \right) \right] \\
q_{NSB} &= q_{\text{Conventional Burnett}} + \frac{\mu^2}{\rho} \left[\frac{15}{4} \frac{1}{\rho RT} \frac{\partial^2}{\partial x^2} \left(k \frac{\partial T}{\partial x} \right) - \frac{15}{4} \frac{1}{\rho^2 R} \frac{\partial \rho}{\partial x} \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) \right. \\
&\quad \left. + 12 \frac{\mu}{\rho RT} \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x^2} - 6 \frac{\mu}{\rho^2 RT} \frac{\partial \rho}{\partial x} \left(\frac{\partial u}{\partial x} \right)^2 \right] \quad (11)
\end{aligned}$$

It is critical to note the differences in the partial derivatives and in the coefficients to the comparable partial derivatives in each of the sets of Burnett relations. Certainly, these variations may lead to observable differences in numerical computations. Numerical experiments must be carried out to evaluate the accuracy of each of the alternate Burnett expressions.

Recall the earlier comparisons of the Material derivative and the inviscid substitutions for the Material derivative through the argon shock. The comparisons showed the stress terms to be similar and the energy terms to be disparate. While the Conventional Burnett stress expression might be adequate, either the MDB or the NSB Burnett heat conduction expressions should provide a more accurate description of the heat conduction through a shock than the analogous Conventional Burnett expression.

It is also important to note that the numerical accuracy of computations made with a given Burnett stress and heat conduction closure model will ultimately determine if any of the alternate forms of the Burnett expressions developed above will actually supplant the Conventional Burnett equations as the most accurate Burnett model. Numerical experimentation, currently accomplished by examining the shock structure of a one-dimensional monatomic gas, is the most established avenue available for comparing different Burnett models. For completeness, we will examine each of the stress models with each of the heat conduction models to compare their physical accuracy and computational stability with the Conventional and Augmented Burnett equations. Table 1 lists the different Burnett stress and heat conduction combinations.

Table 1: Stress and Heat Conduction Combinations

Case	Stress	Heat Conduction	Analysis
1	Navier-Stokes	Navier-Stokes	Completed
2	Conventional Burnett	Conventional Burnett	Completed
3	Augmented Burnett	Augmented Burnett	Completed
4	MDB Burnett	MDB Burnett	In Progress
5	Steady MDB Burnett	Steady MDB Burnett	In Progress
6	NSB Burnett	NSB Burnett	In Progress
7	Conventional Burnett	Steady MDB Burnett	In Progress
8	Conventional Burnett	NSB Burnett	In Progress
9	Steady MDB Burnett	NSB Burnett	In Progress
10	NSB Burnett	Steady MDB Burnett	In Progress

The need for identifying both the unsteady and steady MDB Burnett closure models will become apparent during the subsequent discussion of stability.

Stability of Burnett Equations

Alternate forms of the Burnett constitutive equations have been developed to hopefully improve the accuracy of numerical predictions of gas flow fields, in the continuum transitional flow regime. In order for the modified constitutive equations to be useful, they must be stable to small wavelength disturbances.

Bobylev^[12] in 1982 showed that the linearized Conventional Burnett equations were unstable to small periodic disturbances in a uniform flow field. Fisko showed numerically that the Conventional Burnett equations for a Maxwell gas were unstable for fine grid meshes with spacings smaller than a characteristic mean free path of the fluid. This meant that numerical solutions to high altitude problems would be difficult since the mesh spacing required to capture significant flow features might be smaller than a characteristic mean free path of the fluid.

Zhong was able to develop the Augmented Burnett equations to overcome this deficiency in the Conventional Burnett equations. By adding three Super-Burnett like terms to the set of Conventional Burnett equations, Zhong developed a set of constitutive relations which were stable to a linearized stability analysis, and which were stable numerically to a rigorous numerical test.

The previous sections outlined the development of several alternate forms of the Burnett equations which are currently being investigated. In order to be generally applicable to problems over a wide range of Knudsen Numbers, as is the Augmented Burnett equations, the new forms of the Burnett equations must be shown to be free from instabilities due to small wavelength disturbances. This requires that each of the alternate Burnett equations be stable in both a linearized stability analysis and a rigorous numerical test.

The linearized small disturbance stability analysis provides a necessary though not sufficient condition for stability. If a set of equations is unstable to a linearized stability analysis, then it will be unstable at some small mesh spacing in a numerical computation. If, however, a set of equations is stable to a linearized stability analysis, a rigorous numerical test is still required to prove stability of the non-linear terms of the equations. Since the cost of a linearized stability analysis is small, it is easily used to remove ill-posed sets of equations prior to the more costly and difficult numerical stability test.

The linearized small disturbance stability analysis is carried out in the following manner. A monatomic gas at rest ($u_0 = 0$) with an initial density and temperature ρ_0 and T_0 respectively is perturbed. The non-dimensional perturbation variables

$$\begin{aligned}\rho' &= (\rho - \rho_0)/\rho_0 \\ T' &= (T - T_0)/T_0 \\ u' &= u/\sqrt{RT_0} \\ x' &= x/\lambda_0 \\ t' &= x'/u' \\ \lambda_0 &= \mu_0/\rho_0 \sqrt{RT_0}\end{aligned}$$

are substituted into the conservation equations for mass, momentum and energy:

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) &= 0 \\ \frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + p + \sigma) &= 0 \\ \frac{\partial E}{\partial t} + \frac{\partial}{\partial x}((E + p + \sigma)u + q) &= 0\end{aligned}\tag{12}$$

where

$$p = \rho RT$$

$$E = \rho \left(c_v T + \frac{1}{2} u^2 \right)$$

For a weak disturbance, the following perturbation equations result:

$$\frac{\partial V'}{\partial t} + \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & \frac{2}{3} & 0 \end{bmatrix} \frac{\partial V'}{\partial x} + \frac{\partial}{\partial x} \begin{bmatrix} 0 \\ \sigma' \\ \frac{2}{3} q' \end{bmatrix} = 0 \quad (13)$$

$$\text{where } V' = \begin{bmatrix} \rho' \\ u' \\ T' \end{bmatrix},$$

σ' is the linearized stress and q' the linearized heat conduction. Dropping the prime notation from subsequent expressions, the linearized stress and heat conduction terms for the respective constitutive equations are listed in table 2. The derivative coefficients correspond to a Maxwellian Gas. These coefficients would be somewhat different for other gases such as Argon or Nitrogen.

Table 2: Linearized Constitutive Equations

Constitutive Equations	Linearized Stress	Linearized Heat Conduction
Navier-Stokes	$-\frac{4}{3} u_x$	$-\frac{15}{4} T_x$
Conventional Burnett	$-\frac{4}{3} u_x - \frac{4}{3} \rho_{xx} + \frac{2}{3} T_{xx}$	$-\frac{15}{4} T_x - \frac{7}{4} u_{xx}$
Augmented Burnett	$-\frac{4}{3} u_x - \frac{4}{3} \rho_{xx} + \frac{2}{3} T_{xx} + \frac{2}{9} u_{xxx}$	$-\frac{15}{4} T_x - \frac{7}{4} u_{xx} + \frac{11}{16} T_{xxx} - \frac{5}{8} \rho_{xxx}$
MDB Burnett	$-\frac{4}{3} u_x + \frac{4}{3} u_{xx} + 2 T_{xx}$	$-\frac{15}{4} T_x + \frac{45}{8} T_{xx} + 2 u_{xx}$
Steady MDB Burnett	$-\frac{4}{3} u_x + 2 T_{xx}$	$-\frac{15}{4} T_x + 2 u_{xx}$
NSB Burnett	$-\frac{4}{3} u_x - \frac{4}{3} \rho_{xx} + \frac{2}{3} T_{xx} + \frac{16}{9} u_{xxx}$	$-\frac{15}{4} T_x - \frac{7}{4} u_{xx} + \frac{225}{16} T_{xxx}$

Once the stress and heat conduction terms are specified, a perturbation solution of the form

$$V = V_0 e^{i\alpha x} e^{i\omega t} \quad (14)$$

is defined. Here, ω is the periodic spatial frequency and ϕ controls the time response of the initial perturbations. ϕ can be written as:

$$\phi = \alpha + \beta i$$

where α and β are real numbers and represent the attenuation and dispersion respectively. The solutions to the partial differential equations are stable if the attenuation is not positive.

Substituting the perturbation solution, equation 14, into equation 13 yields a system of algebraic equations of the form:

$$[A] V = 0 \quad (15)$$

The elements of matrix A are a function of the stress and heat conduction models used as constitutive relations.

The characteristic polynomial

$$P(\phi, \omega) = 0 \quad (16)$$

is found from the non-trivial solution to the system of algebraic equations. Table 3 lists the resulting characteristic polynomials for each of the alternate Burnett stress and heat conduction combinations.

Table 3: Characteristic Polynomials For Stress and Heat Conduction Combinations

Case	Stress	Heat Conduction	Characteristic Polynomial
1	Navier-Stokes	Navier-Stokes	$18 \phi^3 + 69 \omega^2 \phi^2 + (30 + 60 \omega^2) \omega^2 \phi + 45 \omega^4$
2	Conventional Burnett	Conventional Burnett	$18 \phi^3 + 69 \omega^2 \phi^2 + (30 + 97 \omega^2 - 14 \omega^4) \omega^2 \phi + 45 \omega^4 + 60 \omega^6$
3	Augmented Burnett	Augmented Burnett	$216 \phi^3 + (828 + 147 \omega^2) \omega^2 \phi^2 + (360 + 1164 \omega^2 + 84 \omega^4 + 22 \omega^6) \omega^2 \phi + 540 \omega^4 + 909 \omega^6 + 72 \omega^8$
4	MDB Burnett	MDB Burnett	$(12 - 61 \omega^2 + 60 \omega^4) \phi^3 + (46 - 100 \omega^2) \omega^2 \phi^2 + (20 - 37 \omega^2 + 32 \omega^4) \omega^2 \phi + 30 \omega^4$
5	Steady MDB Burnett	Steady MDB Burnett	$18 \phi^3 + 69 \omega^2 \phi^2 + (30 + 12 \omega^2 + 48 \omega^4) \omega^2 \phi + 45 \omega^4$
6	NSB Burnett	NSB Burnett	$72 \phi^3 + (276 + 803 \omega^2) \omega^2 \phi^2 + (120 + 388 \omega^2 + 1164 \omega^4 + 1200 \omega^6) \omega^2 \phi + 180 \omega^4 + 915 \omega^6 + 900 \omega^8$
7	Conventional Burnett	Steady MDB Burnett	$18 \phi^3 + 69 \omega^2 \phi^2 + (30 + 52 \omega^2 + 16 \omega^4) \omega^2 \phi + 45 \omega^4 + 60 \omega^6$
8	Conventional Burnett	NSB Burnett	$72 \phi^3 + (276 + 675 \omega^2) \omega^2 \phi^2 + (120 + 388 \omega^2 + 844 \omega^4) \omega^2 \phi + 180 \omega^4 + 915 \omega^6 + 900 \omega^8$
9	Steady MDB Burnett	NSB Burnett	$72 \phi^3 + (276 + 675 \omega^2) \omega^2 \phi^2 + (120 + 292 \omega^2 + 844 \omega^4) \omega^2 \phi + 180 \omega^4 + 675 \omega^6$
10	NSB Burnett	Steady MDB Burnett	$18 \phi^3 + (69 + 32 \omega^2) \omega^2 \phi^2 + (30 + 52 \omega^2 + 96 \omega^4) \omega^2 \phi + 45 \omega^4 + 60 \omega^6$

The time response of the initial perturbation is a function of the periodic frequency. The characteristic polynomial determines the relationship between ϕ and ω . Solutions to the characteristic polynomials of the form $\phi = f(\omega)$ are plotted in figures 3 through 12.

Figures 3 through 5 show the characteristic trajectories of the known constitutive equations. Both the Navier-Stokes and Augmented Burnett equations yield stable characteristic trajectories. The Conventional Burnett equations show branches of the characteristic trajectories which have a positive attenuation, indicating frequencies at which the initial perturbations would grow exponentially.

Figures 6 - 12 show the characteristic trajectories for the alternate forms of the Burnett equations proposed earlier. Notice that two different forms of the MDB Burnett equations are evaluated; the complete MDB Burnett equations and the steady MDB Burnett equations where the time dependent stress and heat conduction terms have been

removed. Figure 6, that of the complete, unsteady MDB Burnett equations, shows characteristic trajectories which exhibit positive attenuation, thereby indicating frequencies at which the MDB Burnett equations are unstable. Each of the other stress and heat conduction combinations yield stable characteristic trajectories, including the steady MDB Burnett relationships. Therefore, cases 7 - 12 listed in table 3 should be explored further for numerical stability and physical accuracy. This exploration will be part of our research program during the next contract year.

Numerical Test of Alternate Burnett Equations

A second order implicit flux-split method^[11,13] has been developed to numerically test the alternate Burnett stress and heat conduction models. The numerical method follows a procedure identical to an earlier successful method for solving the Burnett equations at Stanford. Initial verification of the code has been completed by examining a one-dimensional shock in a Maxwellian gas, using the Conventional and Augmented Burnett equations as constitutive relations. Work now focuses on evaluating the numerical stability of each of the alternate Burnett stress and heat conduction terms.

To date, each of the steady MDB Burnett and NSB Burnett stress and heat conduction terms has been tested for computational stability. All but three of the heat conduction terms have proved to be stable in combination with both the Navier-Stokes and steady MDB or NSB stress and heat conduction terms.

One of the unstable partial derivatives is from the steady MDB heat conduction expression, listed in Eq. (10) The unstable, non-linear term is

$$\frac{45}{8} \frac{\mu^2 u}{\rho T} \frac{\partial^2 T}{\partial x^2}$$

The other two unstable heat conduction terms are from the NSB heat conduction expression listed in Eq. (11). The first partial derivative of the heat conduction expression can be expanded into three terms by using the relationships

$$k = \frac{15}{4} \mu R$$

$$\mu = \mu_o \left(T/T_o \right)^\alpha$$

The expanded partial derivative may be written

$$\frac{15}{4} \frac{\mu^2}{\rho^2 RT} \frac{\partial^2}{\partial x^2} \left(k \frac{\partial T}{\partial x} \right) = \frac{225}{16} \frac{\mu^3 \alpha (\alpha - 1)}{\rho^2 T^3} \left(\frac{\partial T}{\partial x} \right)^3 + \frac{675}{16} \frac{\mu^3 \alpha}{\rho^2 T^2} \frac{\partial T}{\partial x} \frac{\partial^2 T}{\partial x^2} + \frac{225}{16} \frac{\mu^3}{\rho^2 T} \frac{\partial^3 T}{\partial x^3}$$

The first term of the three terms from the expanded first NSB Burnett heat conduction partial derivative is stable. Stable differencing schemes for the other two expanded terms above have not yet been found.

In the near term, alternate methods for developing a stable differencing scheme for the three unstable terms will be examined. Once a method is developed for handling the three partial derivatives, the work effort will shift towards examining each of the alternate stress and heat conduction constitutive relations for physical accuracy. The accuracy of each of the alternate Burnett equations will be presented in future reports as they become available.

Nomenclature

Roman Symbols

c_v	= specific heat
e	= velocity gradient tensor
$\overset{\circ}{e}$	= deviator of the velocity gradient tensor
$\overset{x}{e}$	= trace of the velocity gradient tensor
h	= $1/2kT$
k	= Boltzmann Constant
k	= coefficient of thermal conductivity
K_i	= Burnett Stress Coefficients
m	= molecular mass
p	= ρRT , thermodynamic pressure
$P(\phi, \omega)$	= characteristic polynomial
R	= specific gas constant, k/m
q	= one dimensional heat conduction
\vec{q}	= general heat conduction vector
q'	= linearized, one dimensional heat conduction
T	= translational temperature
T_o	= freestream translational temperature
u	= component of fluid velocity
v	= component of fluid velocity
V'	= vector of perturbation variables, ρ' , u' , T'
w	= component of fluid velocity
x	= spatial coordinate
y	= spatial coordinate
z	= spatial coordinate

Greek Symbols

α	= viscosity-temperature exponent
ϕ	= time response coefficient
λ_o	= characteristic mean free path
μ	= viscosity
μ_o	= freestream viscosity
ρ	= density
σ_{zz}	= viscous stress tensor
σ	= one dimensional viscous stress
$\underline{\sigma}$	= general viscous stress tensor
σ'	= linearized, one dimensional stress
θ_i	= Burnett heat conduction coefficients
ν	= number of molecules per unit volume
ω	= periodic spatial frequency
ω_i	= Burnett stress coefficients
Ω	= fluid rotation, $\frac{1}{2} \nabla \times \vec{v}$

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C. THICK OBLIQUE SHOCK IMPINGING ON COWL LIP

The interaction of a relatively thick oblique shock impinging on the bow shock in front of a blunt leading edge is important in certain high altitude hypersonic flight applications. Such interaction is of special relevance, for example, to thin cowl lips on air-breathing propulsive systems of vehicles such as NASP. To date, the known experiments and computations for this shock interaction phenomena have been restricted to conditions of relatively low altitude flight wherein shock thickness is negligible compared to the normal shock detachment distance. The interaction with relatively thick shock structures is expected to be considerably different, possibly involving, for example, quite different over heating ratios compared to very thin shock structures. Our research will explore this interaction phenomenon for thick shock waves corresponding to flight conditions at high altitude. Initially the Navier-Stokes equations will be used, and subsequently, the Burnett equations. Research on this subject began this past summer.

Our first step has been to determine what the computational boundary conditions should be for properly generating an impinging, relatively thick, hypersonic, oblique-shock structure. Previously, only 1D normal shock structures have been investigated. The desired boundary condition corresponds to specification on an outer computational boundary of the distribution of u and v velocity components that will generate an impinging oblique shock structure of a given strength and thickness. Once generated, interaction with the bow wave on a blunt leading edge can thereby be computed with the usual CFD techniques. It is expected that a very fine grid will be required.

In order to determine how to construct the proper boundary velocity components, an analytical investigation has been made of the governing differential equations for oblique shock structure. Both Navier-Stokes and Burnett equations were considered. A coordinate system was used such that x_n , u_n is the direction and velocity normal, and y_p , v_p the corresponding quantities parallel to the oblique shock. In this coordinate system $\partial/\partial y_p = 0$ for all physical quantities. We find that the continuity equation, the momentum equation

in the x_n direction, and the energy equation all reduce to precisely the same differential equations as for 1D normal shocks. This is the case for both Navier-Stokes and Burnett equations. The momentum equation in the y_p direction, however, does not vanish as it does for 1D normal shocks, and instead becomes an auxiliary equation that must also be satisfied for oblique shocks. With the Navier-Stokes equations this momentum equation for steady state conditions is

$$\rho u_n \frac{\partial v_p}{\partial x_n} = \frac{\partial}{\partial x_n} \left(\mu \frac{\partial v_p}{\partial x_n} \right)$$

One solution is simply $v_p = \text{constant} = v_{p1} = v_{p2}$, where v_{p1} is the parallel velocity component upstream, and v_{p2} is that component downstream, of the oblique shock.

There is, however, another possible solution. The continuity equation shows that ρu_n is constant through the oblique shock, hence the above momentum equation can be integrated once to

$$\rho u_n (v_p - v_{p1}) = \mu \frac{dv_p}{dx_n}$$

and a second integration to

$$(\rho u_n) \int_0^x \frac{dx_n}{\mu} = \ln(v_p - v_{p1}) + \text{constant}$$

Downstream of the shock $\ln(v_p - v_{p1})$ approaches negative infinity, whereas the integral on the left-hand side approaches positive infinity. Thus this type of solution does not satisfy the required boundary conditions; hence the only physically possible solution for Navier-Stokes oblique shock structure is the simple solution of $v_p = \text{constant}$ throughout the shock structure.

With the conventional Burnett equations the auxiliary equation representing momentum conservation in the direction parallel to the shock is

$$\rho u_n (v_p - v_{p1}) = \mu \frac{dv_p}{dx_n} \left[1 - \frac{\mu}{p} \frac{du_n}{dx_n} \left(k_1 - \frac{2k_2}{3} + \frac{k_6}{6} \right) \right]$$

where k_1, k_2, k_6 are the Burnett constants for a given gas. Just as in the case of the Navier-Stokes equations, a solution to this equation is $v_p = \text{constant} = v_{p1} = v_{p2}$. The Burnett terms within the square brackets above are smaller than unity. Hence the term in square

brackets is always positive and upon integration, the same impossibility is encountered as for the Navier-Stokes equations.

$$\ln(v_p - v_{p1}) = \int_0^x \frac{dx_n}{\mu \left[1 - \frac{\mu}{p} \frac{du_n}{dx_n} \left(k_1 - \frac{2k_2}{3} + \frac{k_3}{6} \right) \right]} + \text{constant}$$

Downstream of the shock the left side approaches negative infinity, while the right side approaches positive infinity, which does not satisfy the required boundary condition.

We conclude, therefore, that the only physically possible solution for oblique shock structure is $v_p = \text{constant} = v_{p1} = v_{p2}$ for both the Burnett equations and the Navier-Stokes equations. This result greatly simplifies the construction of appropriate velocity boundary conditions that will generate a thick impinging oblique shock wave. For any desired strength of oblique wave, as represented by the thermodynamic jump conditions across that wave, a 1D normal shock structure is first computed for these same jump conditions and for a Mach number equal to that normal to the oblique wave. Then vector addition of an appropriate constant velocity component in the direction parallel to the oblique wave completes the generation of the desired shock structure for the impinging oblique wave.

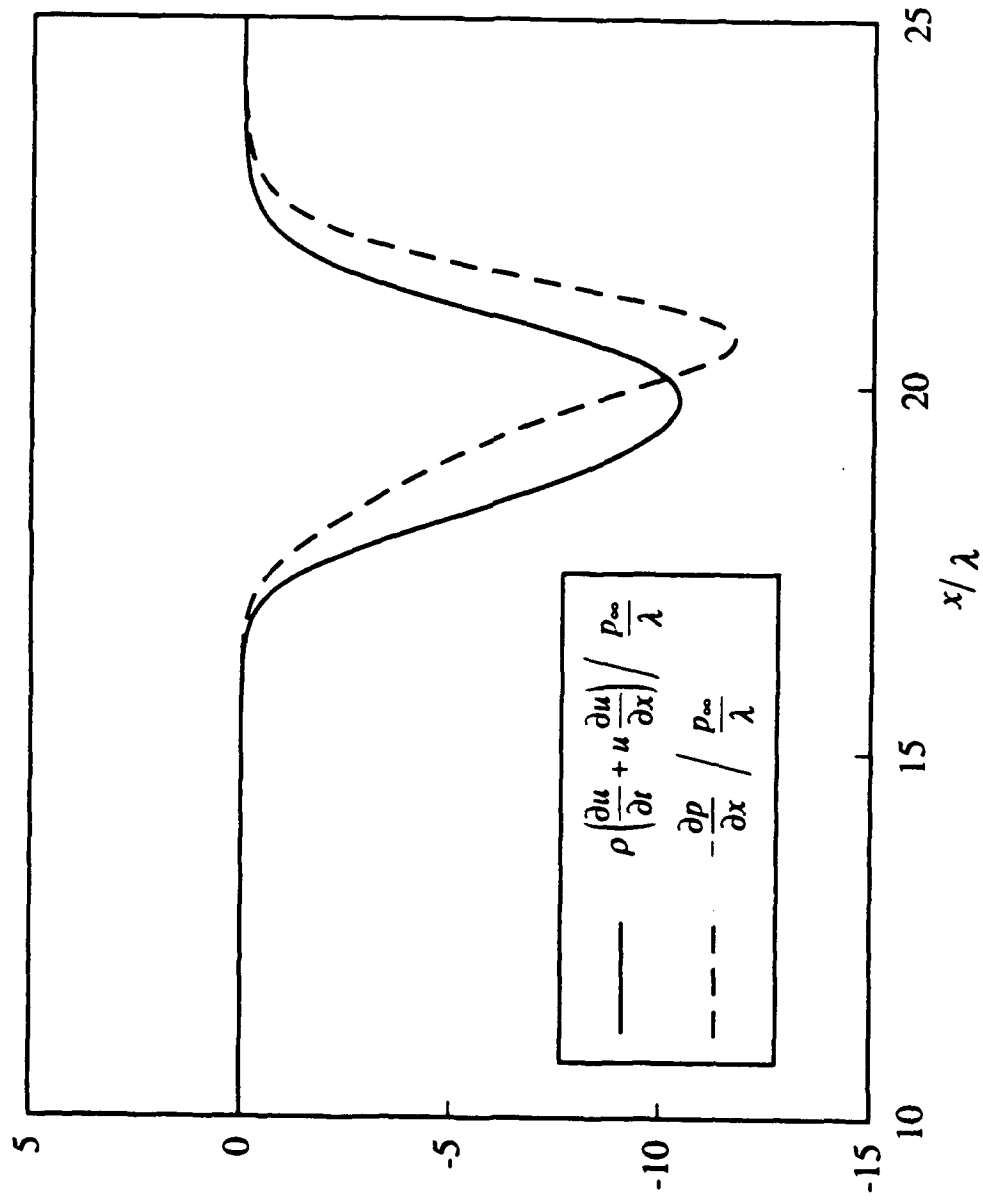


Figure 1: Comparison of Momentum Equation Terms In Mach 5 Argon Shock

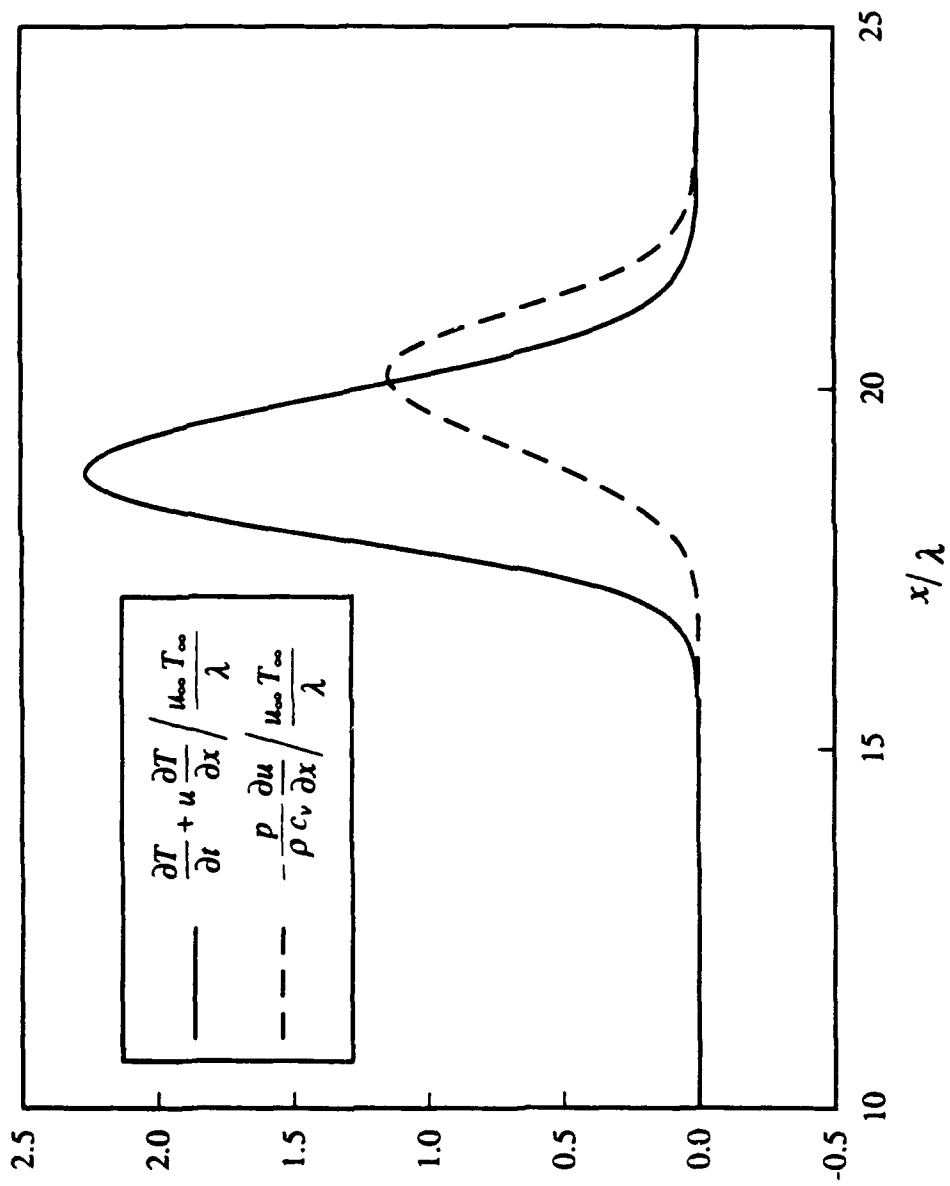


Figure 2: Comparison of Energy Equation Terms In Mach 5 Argon Shock

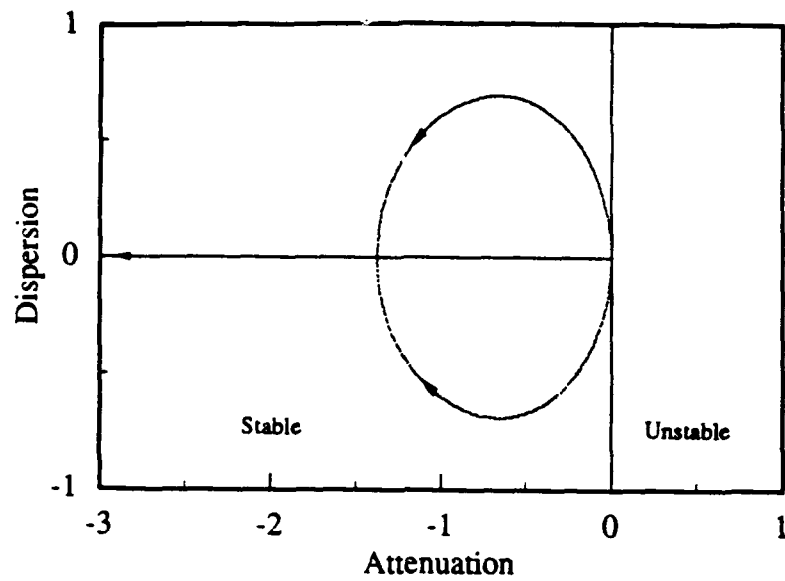


Figure 3: Stability of Perturbation Equations Using Navier-Stokes Stress and Heat Conduction.

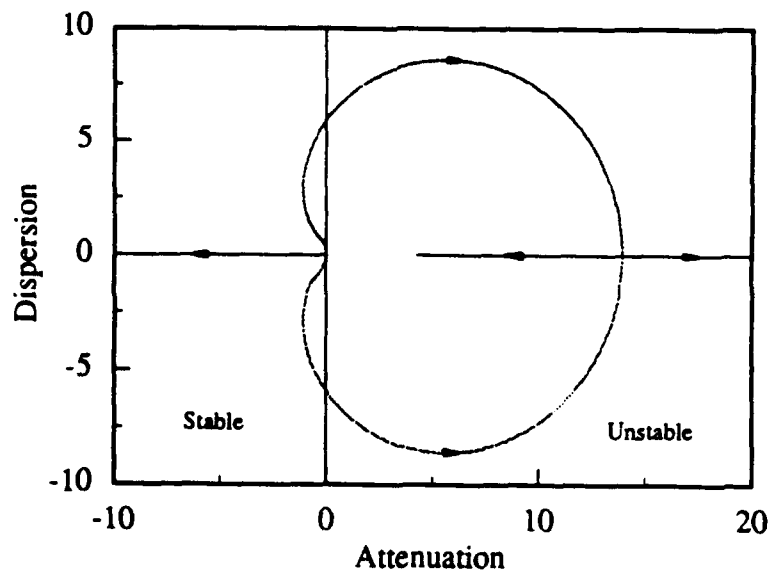


Figure 4: Stability of Perturbation Equations Using Conventional Burnett Stress and Heat Conduction.

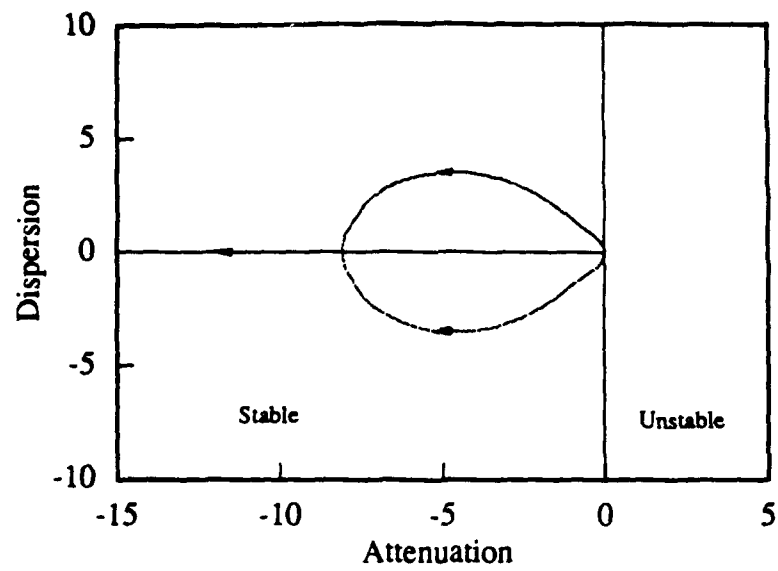


Figure 5: Stability of Perturbation Equations Using Augmented Burnett Stress and Heat Conduction.

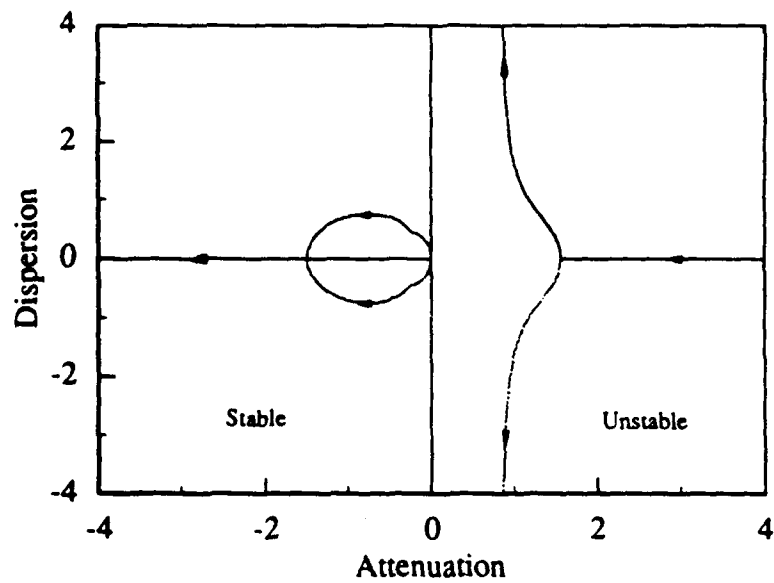


Figure 6: Stability of Perturbation Equations Using Unsteady MDB Burnett Stress and Heat Conduction.

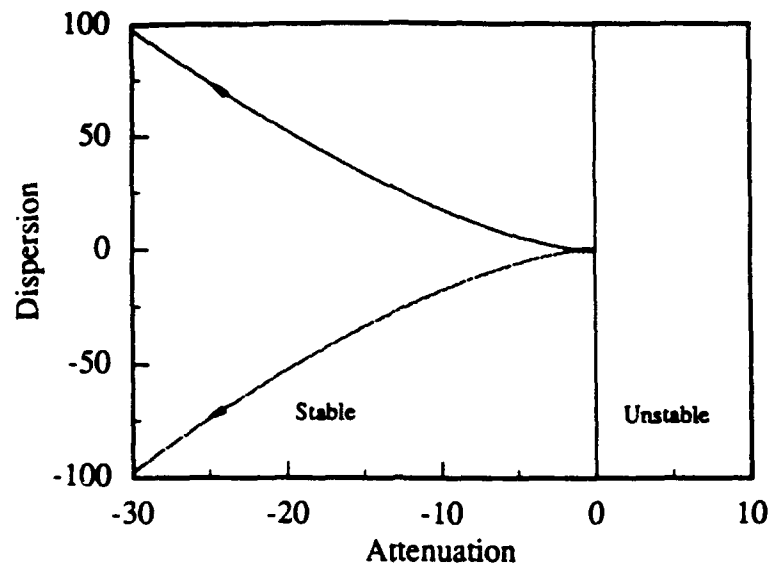


Figure 7: Stability of Perturbation Equations Using Steady MDB Burnett Stress and Heat Conduction.

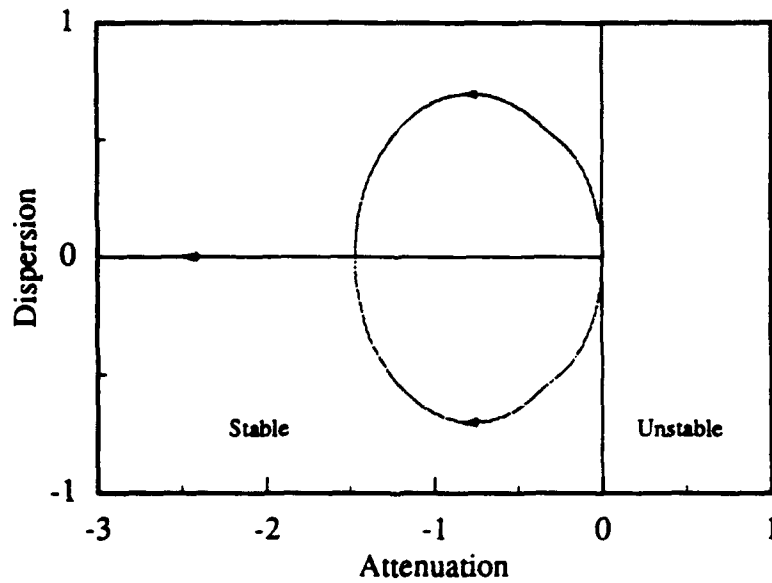


Figure 8: Stability of Perturbation Equations Using NSB Burnett Stress and Heat Conduction.

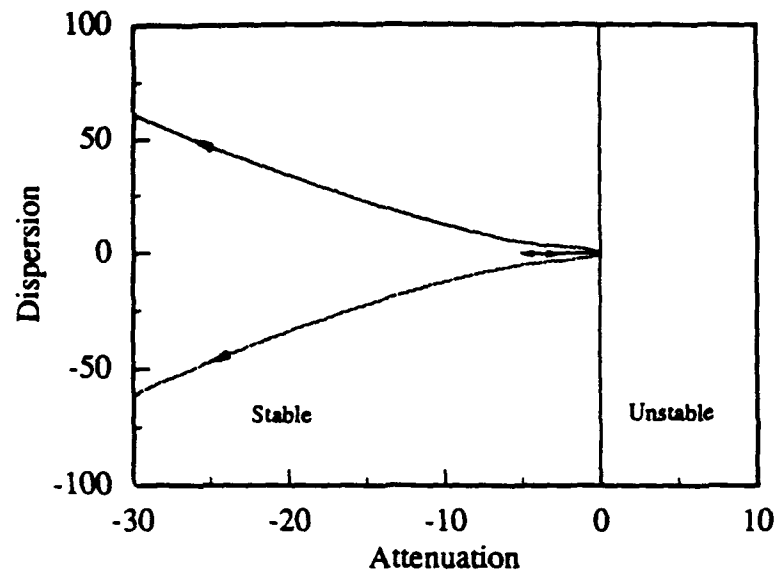


Figure 9: Stability of Perturbation Equations Using Conventional Burnett Stress and Steady MDB Burnett Heat Conduction.

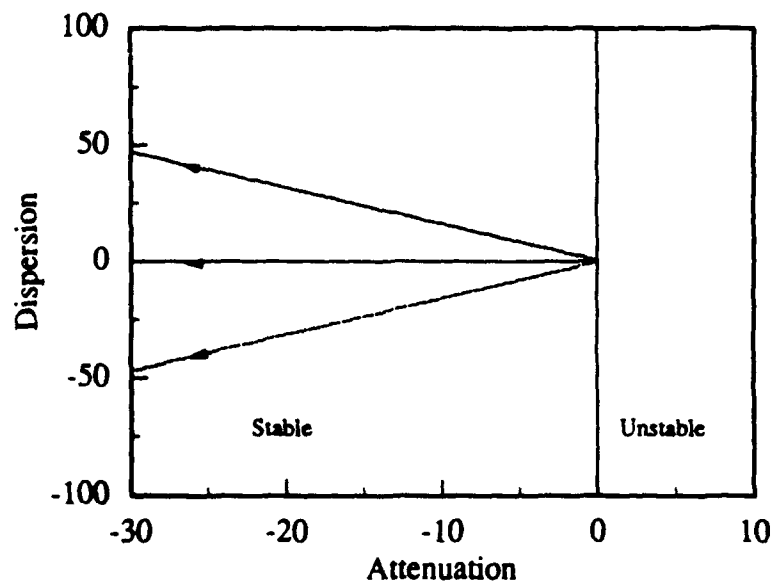


Figure 10: Stability of Perturbation Equations Using Conventional Burnett Stress and NSB Burnett Heat Conduction.

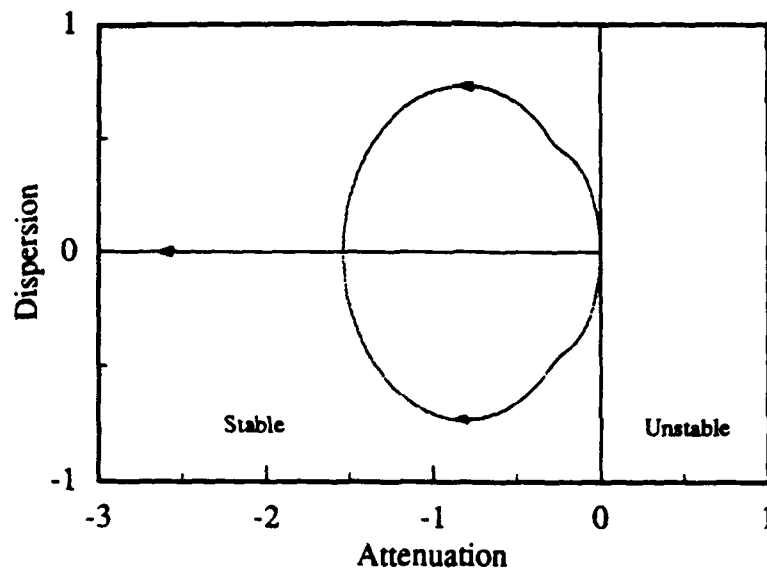


Figure 11: Stability of Perturbation Equations Using Steady MDB Burnett Stress and NSB Burnett Heat Conduction.

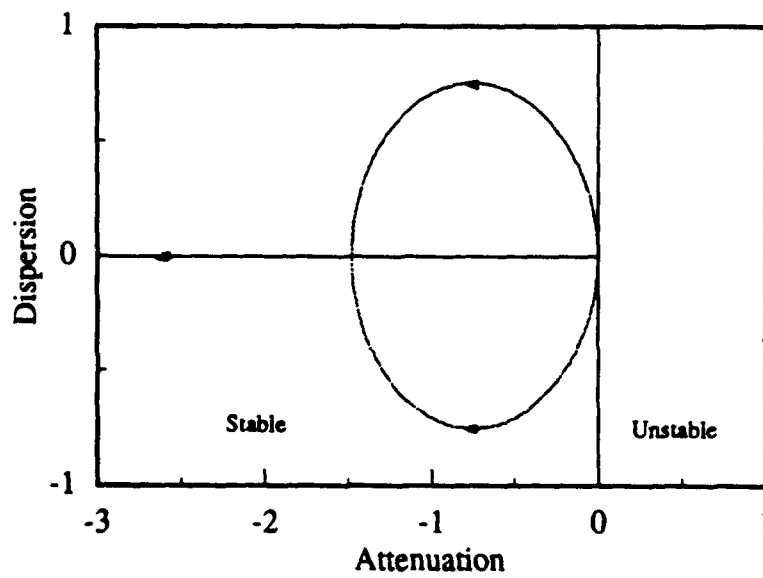
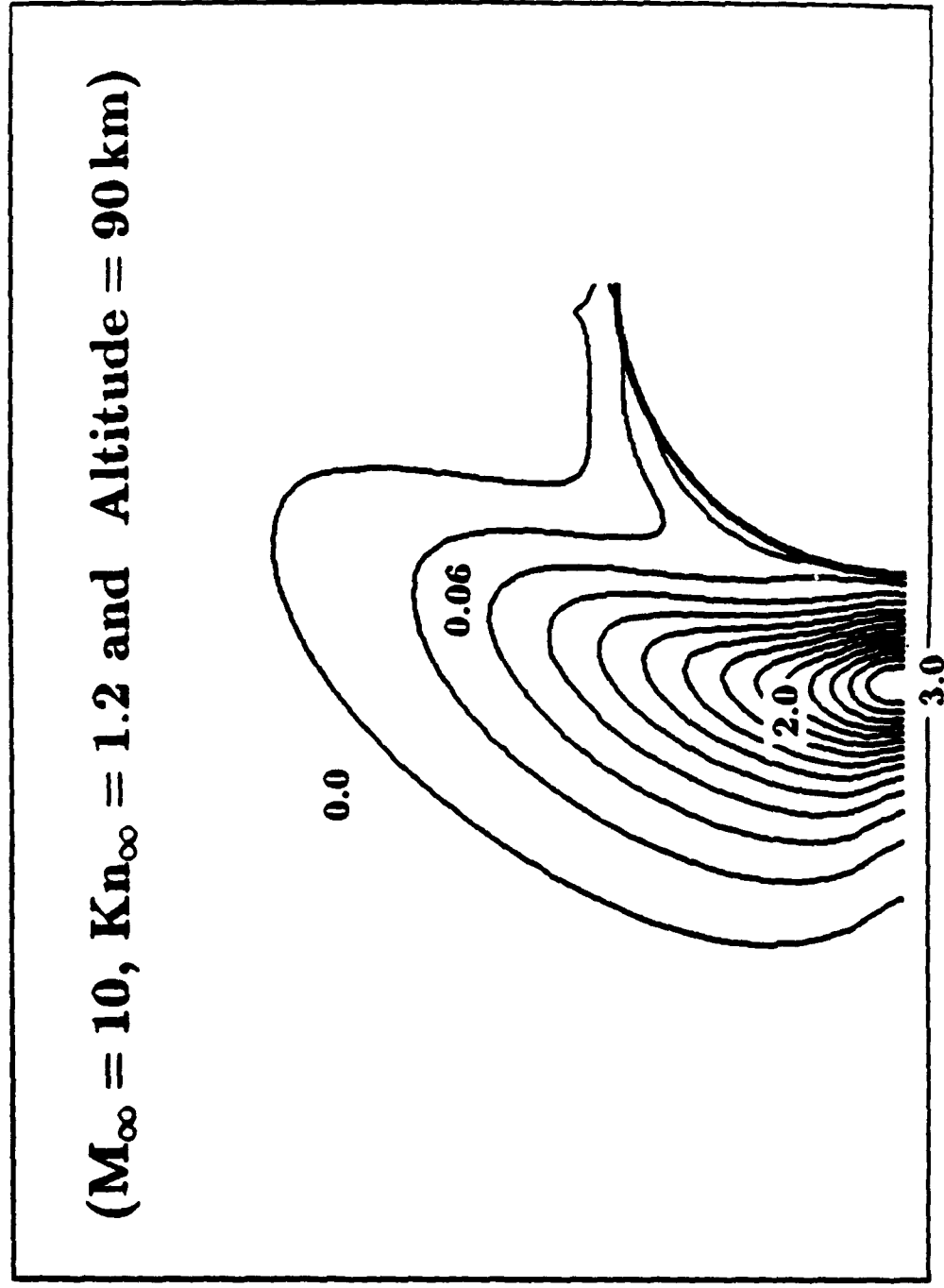


Figure 12: Stability of Perturbation Equations Using NSB Burnett Stress and Steady MDB Burnett Heat Conduction.

CONTOURS OF BURNETT DISSIPATION FUNCTION



DISSIPATION FUNCTION : $-\sum \sigma_{ij} \epsilon_{ij}$

Figure 13